GCSE Maths – Number

Rounding and Limits of Accuracy

Notes

WORKSHEET

Rounding

Numbers with several digits may be **rounded** to the **nearest** 10, 100 or another factor of 10. Rounding **loses some accuracy** because we are **approximating** the number, but it is often more practical. For example, the price of 1 paperclip may be 0.89 pence. We may **round** the price up to 1 pence instead, to make it easier to pay.

General rounding rules:

- If the number you are rounding is followed by 0, 1, 2, 3, or 4: round the number down.
- If the number you are rounding is followed by 5, 6, 7, 8, or 9: round the number up.

How we round numbers depends on the question being asked. We may be asked to round to the nearest 1, 10, 100, 1000, or another number. The principle is the same in each case:

- Take the number 46. If we want to round to the **nearest 10**, we identify the column that contains the '**10s**'.
    - In this case, the 4 represents 40, which is the tens column.
- Now look at the number to the **right of the tens column**. If this **number is less than 5**, then we round down (to 40). If the number is **5 or more**, then we **round up** to 50.
    - For 46, the '6' next to the tens column means that we round up to 50. Therefore, 46 rounded to the nearest ten is 50.

Example: What is 649 rounded to the nearest hundred?
First, we identify the number in the hundreds column. Here, it is 6, representing 600. *Then, look at the number to the right of the 6, which is 4.* *If the number is below 5, then we round down. As 4 is less than 5, we round down to 600.*

Example: What is 0.756 rounded to the nearest 0.1?
Even with decimals, we follow the same approach to rounding. *Identify which number represents the **tenths**. Here, it is 7, which represents 7 tenths or 0.7.* *Looking at the number to the right, it is 5.* *When this number is 5 or more, we **round up**.* *Therefore, 0.756 rounded to the nearest 0.1 is 0.8.*

Decimal Places

A question may specify the **number of decimal places** (d.p.) that are required in the answer. This means that we will need to **round the digit** that is to the **right** of the number of decimal places required.

For example, we may want to round the decimal 3.6783 to two decimal places:

- We count two digits from the decimal point and look at the **next digit to the right**.
  - If this number is less than 5, then we leave the other digits as they are. However, if the next digit is 5 or more, then we will need to **add 1** to the **second digit** from the decimal point.

  This is the case here: the third digit is 8, which is more than 5. So, we then add 1 to the second digit, which is 7. This gives us **3.68**.

Example: What is 135.72572 to 3 decimal places?
As we are looking to round to 3 decimal places, we ***count 3 digits*** *from the decimal point:* **135.725**72 *We then look at the fourth digit, which is 7. This is* ***more than 5****, so we can add 1 to the digit before, making it 6.* *This gives us* ***135.726****.*

Significant Figures

We can also round to a certain number of **significant figures**. Significant figures can be used for all numbers, not just decimals. To use significant figures, we need to ignore any leading zeros. This means that the first significant figure of 0.003 is 3 and not 0.

For example, say we want to round the number 5821 to 2 significant figures.

1. We start at the **first digit** and check if it is 0 or not. If it is not 0, as with the number 5821, then this digit is the **first significant figure**.
2. We can then look to the next digit, 8, which is the second significant figure.
3. To round to 2 significant figures, we need to check the digit **next to the 2nd significant figure** to see if the 2nd significant figure should be **rounded**. For 5821, the third digit is 2, which means that we round the second significant figure, 8, down. To finish writing the number, we put **zeros** in the places after the significant figures. Therefore, 5821 rounded to 2 significant figures is **5800**.

We don't always ignore the zeros when using significant figures.

As an example, let's round the number 70064 to 3 significant figures.

1. We start by looking for the first **non-zero digit**, which is 7 here. Now that we have the first non-zero digit, any zeros after this **are counted as significant figures**. Therefore, the two zeros after 7 are the second and third significant figures.
2. Before writing the answer, we need to check whether we round the last significant figure up or down. Rounding 70064 to 3 significant figures means we **round the third significant figure up** to 1 and write zeros in the remaining places. This leaves us with **70100**.

Example: What is 406829 to 4 significant figures?

*We look for the first **non-zero digit**, which is 4 here. This is our first significant figure.*

*We can now count the **next 3 digits** in the number as the next 3 significant figures.*

1st significant figure: 4
2nd significant figure: 0
3rd significant figure: 6
4th significant figure: 8

*Before writing the answer, we need to check how the last significant figure, 8, should be **rounded**. Since the following digit is 2, we **round down**.*

*We then **put zeros in the remaining places** after the four significant figures.*

*This gives us **406800**.*

When using significant figures with decimals, the approach is almost the same. The only difference is that we **do not write any zeros after the significant figures**. Any zeros **between** the **decimal point** and the **first significant figure** must still be written.

Example: What is 0.00006391 to 3 significant figures?

With decimals, we may have to look further to find the first non-zero digit. Remember, you ignore any leading zeros when identifying the first significant figure.

*Identify the first **non-zero digit**, which is 6 here. This becomes our first significant figure. The next 2 digits are the second and third significant figures.*

1st significant figure: 6
2nd significant figure: 3
3rd significant figure: 9

*Check whether the final significant figure should be **rounded up** or **down** by looking at the digit next to it. Here, it is 1, so we round down.*

*The only difference when writing decimals is that we **do not write any zeros after the significant figures**.*

*The answer is **0.0000639***

Limits of Accuracy

Rounding is practical but does mean that we **lose accuracy**.

For example, imagine that we are weighing bags of cement, which are rounded to the nearest 10 kg (this is the **degree of accuracy**). A bag that is marked as weighing 50 kg could weigh as **little as 45 kg**, or as **much as 54 kg**, but in both instances, it would still be marked as 50 kg.

The lowest possible value that is rounded up to the estimated value is called the **lower limit of accuracy**. The lowest value that would **round up** to the **next estimated value** is called the **upper limit of accuracy**.

- In the example with the cement, the lower limit of accuracy is **45 kg**.
- The upper limit of accuracy is **55 kg**, because this is the lowest value that would be rounded up to the **next estimated value** (**60kg**).

We then use these upper and lower limits of accuracy to write the **error interval**, which shows how **far away** the actual value could be from the estimated value. This is written with **inequality notation**.

With the cement bags, we may have a bag that is marked as 50 kg. To illustrate the potential error (i.e., what the actual weight of the bag may be), we can show it as:

$$45 \text{ kg} \leq Weight < 55 \text{ kg}$$

The $\leq$ sign means that the weight can be **greater than or equal** to 45 kg.
The $<$ sign means that the weight **must be less** than 55 kg. If it were 55 kg, it would be **rounded up** to the next multiple of 10 (60 kg).

An easy method of calculating the upper and lower limits of accuracy is to halve the degree of accuracy (10 kg in this case). We can then **subtract** this from the estimated value to get the lower limit, and **add** it to the estimated value to get the upper limit - but make sure to put the correct signs in.

Example: The heights of sunflowers are marked to the nearest 5 cm. What is the error interval for a sunflower marked as 30 cm?

The degree of accuracy here is 5 cm, so halving that gives us 2.5 cm.

The estimated value is 30 cm.

Lower limit: $30 \text{ cm} - 2.5 \text{ cm} = 27.5 \text{ cm}$

Upper limit: $30 \text{ cm} + 2.5 \text{ cm} = 32.5 \text{ cm}$

$$27.5 \text{ cm} \leq Height < 32.5 \text{ cm}$$

Upper and Lower Bounds (Higher Only)

When we use rounded numbers in calculations, the answer will have an **error interval**. To represent the error interval of the answer, we use **upper** and **lower bounds**. This requires working out the **lowest** and **highest possible values** the rounded numbers could take and performing the calculation with these values.

Going back to the bags of cement, we may need multiple bags. If we want to buy 5 bags marked as weighing 50 kg each, we can calculate the upper and lower bounds for the mass of cement as follows:

- We know that the **degree of accuracy** is 10 kg. Halving this gives us 5 kg, which can be **added** and **subtracted** from 50 kg to give the **upper** and **lower bounds** respectively.

$$\begin{aligned} Lower\ bound &= 45\ \text{kg} \\ Upper\ bound &= 55\ \text{kg} \end{aligned}$$

- We are buying 5 bags.
  - The total lowest mass of cement (lower bound) that we could have is:

$$5 \times 45\ \text{kg} = 225\ \text{kg}$$

  - The total greatest mass of cement (upper bound) that we could have is:

$$5 \times 55\ \text{kg} = 275\ \text{kg}$$

Example: A piece of wood is 50 cm long and 15 cm wide, both measured to the nearest 5 cm. What are the upper and lower bounds for the area of this piece of wood?

The ***degree of accuracy*** *is 5 cm. Halving this gives us 2.5 cm which can be used to obtain the lower and upper bounds for the length and width.*

Length:

$$\begin{aligned} Lower\ bound &= 47.5\ \text{cm} \\ Upper\ bound &= 52.5\ \text{cm} \end{aligned}$$

Width:

$$\begin{aligned} Lower\ bound &= 12.5\ \text{cm} \\ Upper\ bound &= 17.5\ \text{cm} \end{aligned}$$

To calculate the smallest possible area (lower bound), we need to multiply the ***lower bounds*** *of the length and width:*

$$47.5 \times 12.5 = \mathbf{593.75\ cm^2}$$

To calculate the largest possible area (upper bound), we multiply the ***upper bounds*** *of the length and width:*

$$52.5 \times 17.5 = \mathbf{918.75\ cm^2}$$

Rounding and Limits of Accuracy - Practice Questions

1. Round 28812 to the nearest 100.

2. Round 67.89 to the nearest 1.

3. Round 3.058 to 2 decimal places.

4. Round 76329 to 2 significant figures.

5. Round 0.02976 to 3 significant figures.

6. In a car factory, the cars are weighed and marked to the nearest 10kg. What is the error interval for a car marked as weighing 1900kg?

7. What is the error interval for a tennis ball weighing 56g, marked to the nearest 1g?

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.